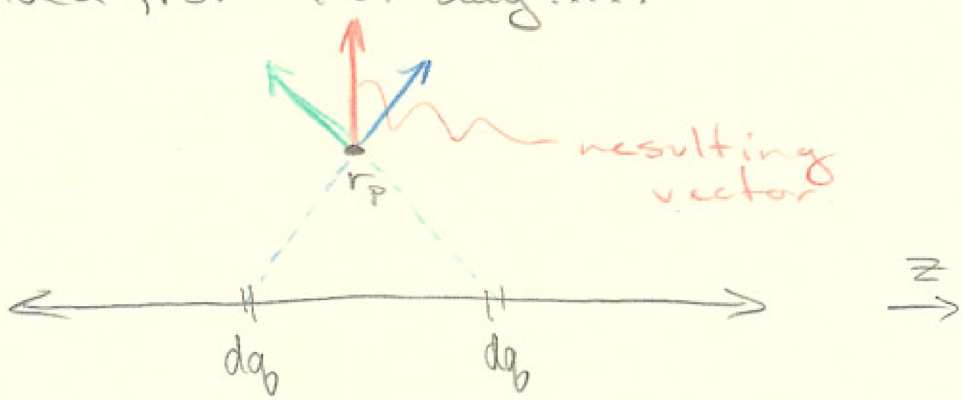
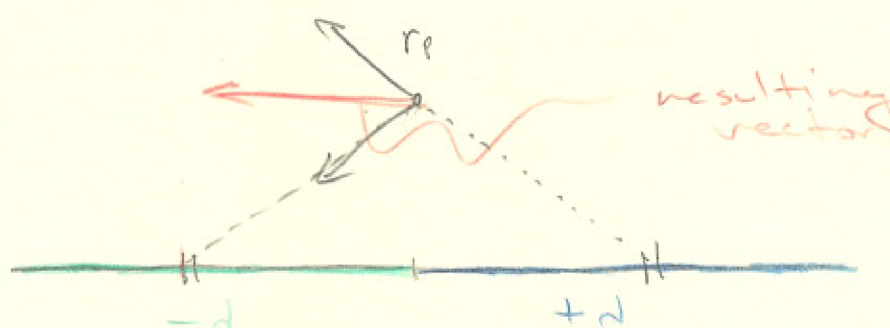
Continued from last day.....



$$|d\vec{E}| = \frac{dq}{4\pi\varepsilon_0}\frac{1}{|\vec{r_p}-\vec{r_s}|^2}$$

Ex:



Surface charge density

Suppose we have a surface with a surface charge density $\sigma(\vec{r_s})$. Break this into small area's (dA). The area at $\vec{r_s}$ causes a small ammount of charge

$$dq = \sigma(\vec{r_s})\, dA_s$$

$$d\vec{E}(\vec{r_p}) = \frac{dq}{4\pi\varepsilon_0}\frac{\vec{r_p}-\vec{r_s}}{|\vec{r_p}-\vec{r_s}|^3}$$

Ex Punctured disc with inner radius 'a' and outer radius 'b'. The disc is centered at the origin and in the plane $z=0$. The disc carries a constant surface charge σ_0

Find $\vec{E}$ @ $r_p = \langle 0, 0, z_p \rangle$

Sol

$$x_s = r_s \cos\theta_s$$

$$y_s = r_s \sin\theta_s$$

Pick any small area @ $(x_s, y_s, 0)$

$$dA_s = r_s\, dr_s\, d\theta_s$$

It will contain

$$dq_s = \sigma_0 r_s\, dr_s\, d\theta_s$$

And will produce a small feild

$$d\vec{E} = \frac{dq_s}{4\pi\varepsilon_0} \frac{\vec{r_p} - \vec{r_s}}{|\vec{r_p} - \vec{r_s}|^3}$$

$$\vec{r_p} = \langle 0, 0, z_p \rangle$$

$$\vec{r_s} = \langle r_s\cos\theta_s, r_s\sin\theta_s, 0 \rangle$$

$$\vec{r_p} - \vec{r_s} = \langle -r_s\cos\theta_s, -r_s\sin\theta_s, z_p \rangle$$

$$|\vec{r_p} - \vec{r_s}| = \sqrt{r_s^2 + z_p^2}$$

$$d\vec{E} = \frac{\sigma_0 r_s\, dr_s\, d\theta_s}{4\pi\varepsilon_0} \frac{z_p\hat{z} - r_s\cos\theta_s\,\hat{x} - r_s\sin\theta_s\,\hat{y}}{(z_p^2 + r_s^2)^{3/2}}$$

Solving the double integral

$$\left.\begin{aligned} E_x &= 0 \\ E_y &= 0 \end{aligned}\right\} \text{b/c sym}$$

$$E_z = \frac{\sigma_0 z_p}{4\pi\varepsilon_0} \int_a^b \frac{r_s\,dr_s}{(z_p^2 + r_s^2)^{3/2}} \int_0^{2\pi} d\theta$$

$$E_z = \frac{\sigma_0 z_p}{2\varepsilon_0} \left[\frac{1}{\sqrt{a^2 + z_p^2}} - \frac{1}{\sqrt{b^2 + z_p^2}} \right]$$